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ELASTIC FIELD CORRELATION FUNCTIONS FOR THE QUASI-ISOTROPIC COMPOSITE MATERIALS UNDER ANISOTROPIC STRAIN

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Binary correlation functions of the stress and strain tensors are obtained for composite materials, the components of which are isotropic. It is assumed that the averaged strain field is arbitrarily anisotropic. An expansion for the elastic field correlation functions in terms of the products of the second rank unit tensor, the average strain tensor and the direction cosines defining the orientation of the straight line connecting the points between which the correlation is sought, is obtained in an explicit form. The stress and strain dispersions are computed. It is shown that the corresponding fourth rank tensors are isotropic under volume strain, have tetragonal symmetry under pure shear, and transversal isotropy under tension. Numerical estimates are obtained for a material, each phase of which has the same ratio of the volume to shear moduli of elasticity equal to $^8/_3$.

1. In [1] we computed second order correlation functions for the stress and strain fields of quasi-isotropic solids such as single phase polycrystals and composite materials, in the isotropic approximation. The physical sense of the isotropic approximation can be expressed by the fact that the contraction of the dispersion of the elasticity coefficients tensor with the mean strain tensors $\langle \varepsilon_{ij} \rangle$ is assumed to have the form (which is, strictly speaking, valid only for the isotropic fields $\langle \varepsilon_{ij} \rangle$)

$$\langle \lambda'_{ijpq} \lambda'_{klmn} \rangle \langle \varepsilon_{pq} \rangle \langle \varepsilon_{mn} \rangle = 3F_1 V_{ijkl} + 2F_2 D_{ijkl} \tag{1.1}$$

$$I_{ijkl} = {}^1/_2 (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \qquad V_{ijkl} = {}^1/_3 \delta_{ij}\delta_{kl}$$

$$D_{ijkl} = I_{ijkl} - V_{ijkl}$$

Here V_{ijkl} and D_{ijkl} denote the volume and deviator components of the fourth rank unit tensor I_{ijkl}.

Below we shall show that the isotropic approximation yields the correct values of the volume and deviator contractions of the tensor of dispersion of the stress and strain fields. However, the approximating assumption that the macroscopic field is isotropic and homogeneous, limits sharply the domain of applicability of the results obtained. For this reason generalization of the computing model becomes imperative. The inhomogeneity of the macroscopic field can be accounted for by using e. g. a scheme proposed by Novozhilov in [2]. Below we show how the anisotropy of the macroscopic elastic field may be taken into account. Since the computations become very cumbersome when an arbitrary microinhomogeneous medium is considered, we shall confine ourselves to the case when each of the phases may be assumed isotropic.

Such properties can be found not only in glass reinforced plastics, but also in such systems as metal-metal, metal-polymer, metal-ceramic, etc., in which the anisotropy of the metallic phase is small either because it is small for particular metal (e. g. the anisotropy coefficient for tungsten is 1.0 and for aluminium it is 1.2), or because each phase represents a nonoriented polycrystal in which the grain size is much smaller than the region of inhomogeneity. In this case the small scale spatial fluctuations of the elastic field related to the property of polycrystallinity can be ignored and only the large scale fluctuations connected with the presence of many phases in the composite, taken into account.

2. The complete expression for the random component of the strain field ε_{ij}' can be written out, using the Green's tensor G_{kl} of the equation of equilibrium, for a medium with averaged moduli of elasticity

$$\varepsilon_{ij}' = G_{(i)k,l(j} * \lambda_{klmn}' \varepsilon_{mn} \tag{2.1}$$

Here the asterisk denotes the operation of contraction, and symmetrization is performed over the indices contained within the brackets. If the macroscopic field is homogeneous, then the integral Fourier transform

$$\varphi^* (\mathbf{k}) \equiv \int \varphi (\mathbf{r}) e^{-i\mathbf{kr}} d\mathbf{r} \tag{2.2}$$

can be used to reduce (1.1) to the form

$$\varepsilon_{ij}'^* = g_{ijkl}^* \lambda_{kl}'^* \tag{2.3}$$

$$\langle\mu\rangle g_{ijkl}^* = \varkappa \nu_{ijkl} - \delta_{i)(k}\nu_{l)(j}, \quad \varkappa = \langle 3K + \mu\rangle / \langle 3K + 4\mu\rangle$$

$$\nu_{ij\ldots l} \equiv \nu_i \nu_j \ldots \nu_l, \quad \nu_i = k_i / k, \quad \lambda_{ij}^* = K^* \varepsilon \delta_{ij} + 2\mu^* e_{ij} \tag{2.4}$$

$$\varepsilon \equiv \langle \varepsilon_{ii}\rangle, \qquad e_{ij} \equiv \langle \varepsilon_{ij} - {}^1/_3 \varepsilon_{kk} \delta_{ij}\rangle$$

Here the terms $\lambda_{klmn}' \varepsilon_{mn}'$ of the second order of smallness have been omitted. From (2.3) and (2.4) we find

$$-\varepsilon_{ij}'^* = \xi'^* \nu_{ij} + 2\eta'^* \beta_{ij} \tag{2.5}$$

$$\xi^* = \chi^* - 2\varkappa\eta^*\beta, \quad \chi = \frac{3K\varepsilon}{\langle 3K + 4\mu\rangle}, \quad \eta = \frac{\mu}{\langle\mu\rangle}$$
$$\beta_{ij} \equiv \nu_{(i} e_{j)k} \nu_k, \qquad \beta \equiv \beta_{ii} \tag{2.6}$$

The random component of the stress field is found in the same manner. We have, within the previous approximation,

$$\sigma_{ij}' = \lambda_{ijkl}' \langle \varepsilon_{kl}\rangle + \langle \lambda_{ijkl}\rangle \varepsilon_{kl}' \tag{2.7}$$

from which using the expression (2.5) we find the Fourier transform of the random component of the stress field

$$\sigma_{ij}'^* = 2\langle\mu\rangle [\xi'^* (\delta_{ij} - \nu_{ij}) + \eta'^* (\beta\delta_{ij} + e_{ij} - 2\beta_{ij})] \tag{2.8}$$

Expressions (2.5) and (2.8) enable us to find the correlation functions for the stress and strain fields. To do this we shall use the following equations:

$$S_{ijkl} (\mathbf{r}) \equiv \langle \sigma_{ij}' (\mathbf{r} + \mathbf{r}_1) \sigma_{kl}' (\mathbf{r}_1)\rangle$$
$$E_{ijkl}(\mathbf{r}) \equiv \langle \varepsilon_{ij}' (\mathbf{r} + \mathbf{r}_1) \varepsilon_{kl}' (\mathbf{r}_1)\rangle \tag{2.9}$$

$$\langle \sigma_{ij}'^* (\mathbf{k} + \mathbf{k}_1) \bar{\sigma}_{kl}'^* (\mathbf{k}_1)\rangle = 8\pi^3 \delta (\mathbf{k}) S_{ijkl}^* (\mathbf{k}_1)$$
$$\langle \varepsilon_{ij}'^* (\mathbf{k} + \mathbf{k}_1) \bar{\varepsilon}_{kl}'^* (\mathbf{k}_1)\rangle = 8\pi^3 \delta (\mathbf{k}) E_{ijkl}^* (\mathbf{k}_1) \tag{2.10}$$

Here the angle brackets denote statistical averaging and the prime accompanying a letter denotes a complex conjugate. The statistical homogeneity of the elastic fields which average to a constant value, is also taken into account.

We have analogous relations for the correlation function of the tensor of the moduli of elasticity

$$\Lambda_{pqrs}^{ijkl}(\mathbf{r}) \equiv \langle \lambda_{ijkl}'(\mathbf{r}+\mathbf{r}_1)\,\lambda_{pqrs}'(\mathbf{r}_1)\rangle \tag{2.11}$$

$$\langle \lambda_{ijkl}'^{*}(\mathbf{k}+\mathbf{k}_1)\,\overline{\lambda}_{pqrs}'^{*}(\mathbf{k}_1)\rangle = 8\pi^3\delta(\mathbf{k})\,\Lambda_{pqrs}^{ijkl}(0)\,\varphi^*(\mathbf{k}_1)$$

Here we use the hypothesis of separation of the tensor and coordinate relationships in the binary correlation function of the tensor of the moduli of elasticity [1]. Inserting into (2.10) the explicit expressions for $\sigma_{ij}'^{*}(\mathbf{k})$ and $\varepsilon_{ij}'^{*}(\mathbf{k})$ in accordance with (2.5) and (2.8) and taking (2.11) into account, we find

$$E_{ijkl}^{*}(\mathbf{k}) = D_{ijkl}^{p}\varphi^*(\mathbf{k}), \qquad S_{ijkl}^{*}(\mathbf{k}) = 4\langle\mu\rangle^2 D_{ijkl}^{q}\varphi^*(\mathbf{k})$$

$$D_{ijkl}^{xy} \equiv \langle x_{ij}' y_{kl}'\rangle = v_1 v_2 (x_{ij}^{(1)} - x_{ij}^{(2)})(y_{kl}^{(1)} - y_{kl}^{(2)})$$

$$D_{ijkl}^{x} \equiv D_{ijkl}^{xx}, \qquad p_{ij} \equiv \xi\nu_{ij} + 2\eta\beta_{ij}$$

$$q_{ij} \equiv \xi(\delta_{ij} - \nu_{ij}) + \eta(\beta\delta_{ij} + e_{ij} - 2\beta_{ij})$$

Here p_{ij} differs from the Fourier transform of the strain $\varepsilon_{ij}^{*}(\mathbf{k})$ in not only having a different sign, but also by the fact that the quantities ξ and η are assumed to be functions of the coordinates, while β_{ij} and β are, as before, taken from the domain of the wave numbers. An analogous statement holds for q_{ij}. Therefore the dispersion of p_{ij} and q_{ij} depends on $\mathbf{k}$. The latter reflects the fact that the coordinate and the tensor relations can be separated from each other only for the correlation function of the moduli of elasticity, it cannot be performed for the stress and strain fields [1, 3].

3. Let us now obtain the inverses of the Fourier transforms of the correlation functions. We introduce the following auxiliary functions

$$J_{ij\ldots l}^{(\alpha)*}(\mathbf{k}) = (-1)^{\alpha}\,\nu_{ij\ldots l}\,\varphi^*(\mathbf{k})$$

$$J_{ij\ldots l}^{(\alpha)}(\mathbf{r}) = \nabla_i\nabla_j\ldots\nabla_l\,\frac{1}{8\pi^3}\int e^{i\mathbf{k}\mathbf{r}}k^{-2\alpha}\varphi^*(\mathbf{k})\,d\mathbf{k} \tag{3.1}$$

Here the order of differentiation is equal to 2α. Using (3.1) we obtain

$$E_{ijkl}(\mathbf{r}) = Q\begin{pmatrix} ij \\ kl \end{pmatrix}[D_\chi J_{ijkl}^{(2)} + 4D_{\chi\eta}(J_{ijkm}^{(2)}e_{ml} + \varkappa J_{ijklmn}^{(3)}e_{mn}) +$$
$$4D_\eta(J_{ikmn}^{(2)}e_{jm}e_{ln} + 2\varkappa J_{ijkmnr}^{(3)}e_{mn}e_{lr} + \varkappa^2 J_{ijklmnrs}^{(4)}e_{mn}e_{rs})]$$

$$(1/4\langle\mu\rangle^2)\,S_{ijkl}(\mathbf{r}) = Q\begin{pmatrix} ij \\ kl \end{pmatrix}\{D_\chi(\delta_{ij}\delta_{kl} + 2\delta_{ij}J_{kl}^{(1)} + J_{ijkl}^{(2)}) + 2D_{\chi\eta}[(\delta_{ij} +$$
$$J_{ij}^{(1)})e_{kl} + (2\varkappa - 1)\delta_{ij}\delta_{kl}J_{mn}^{(1)}e_{mn} + (4\varkappa - 1)\delta_{ij}J_{klmn}^{(2)}e_{mn} +$$
$$2\varkappa J_{ijklmn}^{(3)}e_{mn} + 2(\delta_{ij}J_{km}^{(1)}e_{ml} + J_{ijkm}^{(2)}e_{ml})] + D_\eta[e_{ij}e_{kl} + (2\varkappa - 1)^2 \times$$
$$\delta_{ij}\delta_{kl}J_{mnrs}^{(2)}e_{mn}e_{rs} + 2(2\varkappa - 1)\delta_{ij}J_{mn}^{(1)}e_{kl}e_{mn} + 4\varkappa J_{ijmn}^{(2)}e_{kl}e_{mn} +$$
$$4\varkappa(2\varkappa - 1)\delta_{ij}J_{klmnrs}^{(3)}e_{mn}e_{rs} + 4\varkappa^2 J_{ijklmnrs}^{(4)}e_{mn}e_{rs} + \tag{3.2}$$
$$4(2\varkappa - 1)\delta_{ij}J_{kmnr}^{(2)}e_{mn}e_{lr} + 8\varkappa J_{ijkmnr}^{(3)}e_{mn}e_{lr} + 4J_{im}^{(1)}e_{jm}e_{kl} + 4J_{ikmn}^{(2)}e_{jm}e_{nl}]\}$$

Here $Q\binom{ij}{kl}$ is the symmetrization operator performed within a pair of indices and between them

$$Q\binom{ij}{kl}A_{ijkl} = A_{[(ij)(kl)]} \equiv {}^1\!/_2\,(A_{(ij)(kl)} + A_{(kl)(ij)})$$

The integrals $J_{ij\ldots l}^{(\alpha)}$ given by Eqs. (3.1) can be written in the form of an expansion in terms of the products of δ_{ij} and $n_{ij} = n_i n_j$, where $n_i = x_i / r$

$$J_{ij\ldots kl}^{(\alpha)} \equiv \sum_{\beta=0}^{\alpha} T_\beta^{(\alpha)} \psi_{\beta ij\ldots kl}^{(\alpha)} \tag{3.3}$$

$$\psi_{\beta ij\ldots kl}^{(\alpha)} = \sum_P P\delta_{ij}\ldots\delta_{pq}n_{rs}\ldots n_{kl} \tag{3.4}$$

In (3.4) P denotes the operator of transposition of indices and summation is performed over all possible permutations except the identity permutations. There are $\alpha - \beta$ cofactors of the type δ_{ij} in (3.4) and β cofactors of the type n_{rs}. Thus for $\alpha = 2$ we have

$$\psi_{0ijkl}^{(2)} = \delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$$

$$\psi_{1ijkl}^{(2)} = \delta_{ij}n_{kl} + \delta_{kl}n_{ij} + \delta_{ik}n_{jl} + \delta_{jl}n_{ik} + \delta_{il}n_{jk} + \delta_{jk}n_{il}$$

$$\psi_{2ijkl}^{(2)} = n_{ij}n_{kl}$$

The number of components of the tensor $\psi_{\beta ij\ldots kl}^{(\alpha)}$ is

$$N_{\alpha\beta} = \frac{(2\alpha)!}{2^{\alpha-\beta}(\alpha-\beta)!\,(2\beta)!}$$

The coefficients of the expansion (3.3) obey the following recurrence relations:

$$-T_\beta^{(\alpha)} = T_{\beta-1}^{(\alpha-1)} + [2(\alpha+\beta)-1]\,T_{\beta-1}^{(\alpha)} \tag{3.5}$$

For a nonoriented mechanical mixture of isotropic components the surface of the three-dimensional representation of correlations is spherical and the function $\varphi(\mathbf{r})$ can be chosen in the exponential form

$$\varphi(\mathbf{r}) = \exp(-r/a)$$

With $\varphi(r)$ chosen in this manner, the first five coefficients $T_0^{(\alpha)}$ become

$$T_0^{(0)} \equiv \varphi = \exp(-1/\rho), \qquad \rho = a/r$$

$$T_0^{(1)} = (1 + 2\rho + 2\rho^2)\rho\varphi - 2\rho^3$$

$$T_0^{(2)} = (1 + 5\rho + 12\rho^2 + 12\rho^3)\rho^2\varphi + (1 - 12\rho^2)\rho^3$$

$$T_0^{(3)} = (1 + 9\rho + 39\rho^2 + 90\rho^3 + 90\rho^4)\rho^3\varphi - ({}^1\!/_4 - 6\rho^2 + 90\rho^4)\rho^3$$

$$T_0^{(4)} = (1 + 14\rho + 95\rho^2 + 375\rho^3 + 8\cdot 7!!\rho^5)\rho^4\varphi + ({}^1\!/_{24} - {}^3\!/_2\rho^2 + 3\cdot 5!!\rho^4 - 8\cdot 7!!\rho^6)\rho^3$$

When $r \to 0$, the coefficients $T_0^{(\alpha)}$ assume the following limiting values

$$T_0^{(\alpha)} = (-1)^\alpha/(2\alpha+1)!! \tag{3.6}$$

Inserting the asymptotic values of $T_0^{(\alpha)}$ given by (3.6) into the recurrence relations (3.5) we confirm that when $\beta \neq 0$, the coefficients $T_\beta^{(\alpha)}$ vanish as $r \to 0$. This reflects the condition that the medium is isotropic and according to this condition the expression for the symmetric fourth rank tensor must only contain all possible combinations of the Kronecker deltas δ. When $r \to \infty$, we obtain the following limiting values

$$T_0^{(\alpha)} \approx (-1)^{\alpha} \cdot 2\rho^3/[2(\alpha-1)]!!$$

4. Expressions (3.2) yield the stress and strain dispersions. We find them by setting the argument r equal to zero in the corresponding correlation functions. After the necessary manipulations we obtain

$$15\overset{\circ}{E}_{ijkl} = (D_\chi + {}^8/_{63}\varkappa^2 D_\eta \partial)\,\psi^{(2)}_{0ijkl} + 2(1 - {}^4/_7\varkappa)\,D_{\chi\eta}\varphi^{(2)}_{1ijkl} + 2(1 - {}^8/_7\varkappa + {}^{16}/_{63}\varkappa^2)\,D_\eta\varphi^{(2)}_{2ijkl} + (1 - {}^8/_7\varkappa + {}^{32}/_{63}\varkappa^2)\,D_\eta\zeta^{(2)}_{1ijkl} + 2D_\eta(e_{ij}e_{kl} - \delta_{[ij}\partial_{kl]}) \quad (4.1)$$

$$({}^{15}/_4\langle\mu\rangle^2)S^0_{ijkl} = 15\overset{\circ}{E}_{ijkl} + [5D_\chi + 2(1-2\varkappa)(1 - {}^{10}/_7\varkappa)\,D_\eta\partial]\delta_{ij}\delta_{kl} - 8(1-2\varkappa)\times (1 - {}^4/_7\varkappa)\,D_\eta\delta_{[ij}\partial_{kl]} + 4(4\varkappa - 1)\,D_{\chi\eta}\,\delta_{[ij}e_{kl]} + (8\varkappa - 5)\,D_\eta e_{ij}e_{kl}$$

where

$$\varphi^{(2)}_{1ijkl} = \delta_{ij}e_{kl} + \delta_{ik}e_{jl} + \delta_{jl}e_{ik} + \delta_{kl}e_{ij} + \delta_{il}e_{jk} + \delta_{jk}e_{il}$$

$$\zeta^{(2)}_{1ijkl} = \delta_{ij}\partial_{kl} + \delta_{ik}\partial_{jl} + \delta_{jl}\partial_{ik} + \delta_{il}\partial_{jk} + \delta_{jk}\partial_{il} + \delta_{kl}\partial_{ij}$$

$$\varphi^{(2)}_{2ijkl} = e_{ij}e_{kl} + e_{ik}e_{jl} + e_{il}e_{jk}, \quad \partial_{kl} = e_{kn}e_{nl}, \quad \partial = \partial_{kk}$$

Comparing the expressions for the correlation functions with those for dispersions, we note that in the first case the quantity which we seek can be written in the form of an expansion in terms of the Kronecker deltas δ, averaged deviators of the strain tensor and combinations of the direction cosines defining the orientation of the line connecting two points between which the correlation is sought, in the second case the expansion contains no direction cosines. Because of this the expressions for dispersions are simpler than those for the correlation functions.

Expressions (4.1) become considerably simplified when the averaged strains are pure volume strains, i.e. when $\langle\varepsilon_{ij}\rangle = \varepsilon\delta_{ij}$, while $e_{ij} = \beta_{ij} = 0$. In this case we have

$$\overset{\circ}{E}_{ijkl} = {}^1/_{15}D_\chi\delta_{ijkl}, \qquad \delta_{ijkl} = \psi^{(2)}_{0ijkl}$$

$$\overset{\circ}{S}_{ijkl} = {}^4/_{15}\langle\mu\rangle^2 D_\chi(\delta_{ijkl} + 5\delta_{ij}\delta_{kl})$$

Consider now another particular case of pure shear macrostrains. Setting $\langle\varepsilon_{ij}\rangle = 2e_{12}\delta_{1(i}\delta_{j)2}$, we obtain

$$\overset{\circ}{E}_{ijkl} = {}^4/_{15}e^2_{12}D_\eta d^e_{ijkl}, \quad d^e_{ijkl} = 2(1 - {}^6/_7\varkappa + {}^2/_7\varkappa^2)\,\delta_{ijkl} - \delta_{ij}\delta_{kl} - (1 - {}^{10}/_7\varkappa + {}^8/_{21}\varkappa^2)(\delta_{i3}\delta_{j3}\delta_{kl} + \delta_{ij}\delta_{k3}\delta_{l3}) - (7 - {}^{40}/_7\varkappa + {}^{32}/_{21}\varkappa^2)\,\delta_{3(i}\delta_{j)(k}\delta_{l)3} + 4(1 - {}^6/_7\varkappa + {}^4/_{21}\varkappa^2)(2\delta_{i3}\delta_{i3}\delta_{k3}\delta_{l3} - \Sigma\,\delta_{in}\delta_{jn}\delta_{kn}\delta_{ln}) \quad (4.2)$$

$$\overset{\circ}{S}_{ijkl} = {}^4/_{15}\langle\mu\rangle^2\, e^2_{12}D_\eta d^s_{ijkl}, \quad d^s_{ijkl} = (3 + {}^8/_7\varkappa + {}^{16}/_7\varkappa^2)\,\delta_{ijkl} - (3 + {}^8/_7\varkappa - {}^{16}/_7\varkappa^2)\,\delta_{ij}\delta_{kl} - {}^{32}/_7\varkappa(1 - {}^2/_3\varkappa)(\delta_{i3}\delta_{j3}\delta_{kl} + \delta_{ij}\delta_{k3}\delta_{l3}) - 2(1 + {}^8/_7\varkappa + {}^{16}/_{21}\varkappa^2)\,\delta_{3(i}\delta_{j)(k}\delta_{l)3} + 2(3 + {}^8/_7\varkappa + {}^{32}/_{21}\varkappa^2)\times (2\delta_{i3}\delta_{j3}\delta_{k3}\delta_{l3} - \Sigma\,\delta_{in}\delta_{jn}\delta_{kn}\delta_{ln})$$

The expressions obtained show clearly that if the macrostrain is a pure volume strain, then the stress and strain dispersion tensors are isotropic. For the pure shear strain, these tensors have tetragonal symmetry in the x_1x_2-plane, with the fourth order axis of symmetry directed along the x_3-axis. Thus when the microinhomogeneous medium undergoes volume strain, then each of the stress and strain dispersion tensors has two independent components, the number rising to six in the case of shear strain. The matrix notation is convenient in estimating the signs of the components of the tensors d^e_{ijkl} and d^s_{ijkl}. Each of the matrices d^e_{mn} and d^s_{mn} has six independent components which in accordance with (4.2) are

$$\begin{aligned}
&d^e_{11} = 1 - {}^{12}/_7\varkappa + {}^{20}/_{21}\varkappa^2, && d^e_{33} = {}^4/_{21}\varkappa^2 \\
&d^e_{12} = 1 - {}^{12}/_7\varkappa + {}^4/_7\varkappa^2, && d^e_{13} = -{}^2/_7\varkappa(1 - {}^2/_3\varkappa) \\
&d^e_{44} = 1 - {}^8/_7\varkappa + {}^{16}/_{21}\varkappa^2, && d^e_{66} = 8(1 - {}^6/_7\varkappa + {}^2/_7\varkappa^2) \\
&d^s_{11} = {}^{128}/_{21}\varkappa^2, && d^s_{33} = 4(1 - {}^{24}/_7\varkappa + {}^{64}/_{21}\varkappa^2) \\
&d^s_{12} = {}^{32}/_7\varkappa^2, && d^s_{13} = -{}^{32}/_7\varkappa(1 - {}^5/_3\varkappa) \\
&d^s_{44} = 1 - {}^8/_7\varkappa + {}^{16}/_{21}\varkappa^2, && d^s_{66} = 3 + {}^8/_7\varkappa + {}^{16}/_7\varkappa^2
\end{aligned} \tag{4.3}$$

We see that when $m = n$, d^e_{mn} and d^s_{mn} are positive. This also follows from the definition describing these quantities as squares of deviations from the mean of the corresponding field components. The components $d_{12}{}^s$ are positive for any $\varkappa$, $d_{12}{}^e$ for $\varkappa < {}^3/_2 - 1/\sqrt{2}$ and $d_{13}{}^s$ for $\varkappa > {}^3/_5$. Finally, $d_{13}{}^e$ is negative for any $\varkappa$.

We note that a similar treatment applied to the case of longitudinal strain in a microinhomogeneous medium leads to the conclusion that in this case the tensors $\overset{\circ}{S}_{ijkl}$ and $\overset{\circ}{E}_{ijkl}$ are transversally isotropic (hexagonal symmetry).

Let us estimate the anisotropy of the matrices d^e_{mn} and d^s_{mn} under shear strain, The following relations define the four anisotropy coefficients specifying the deviation of the matrix structures from their isotropic form

$$\begin{aligned}
&A_1 = d_{13} / d_{12}, && A_2 = d_{33} / d_{11} \\
&A_3 = d_{44} / d_{66}, && A_4 = 2d_{66} / (d_{11} - d_{12})
\end{aligned}$$

For most materials the parameter $\varkappa$ varies within the limits $0.7 < \varkappa < 0.8$. Therefore, setting $\varkappa = {}^3/_4$ we find from (4.3) that the anisotropy parameters are equal to -3, ${}^3/_7$, ${}^4/_{29}$ and ${}^{116}/_3$ for $A_\alpha{}^e$ and ${}^1/_3$, ${}^1/_6$, ${}^1/_9$ and 12 for $A_\alpha{}^s$. This shows that the tensors $\overset{\circ}{E}_{ijkl}$ and $\overset{\circ}{S}_{ijkl}$ are essentially anisotropic and the isotropic approximation (1.1) is found to be much too inaccurate for the shear strain. We stress that the discrepancy becomes particularly large for the stress ans strain dispersion in the shear plane, for which the isotropic approximation with $\varkappa = {}^3/_4$ yields values which are, respectively, 3.5 and 13 times smaller.

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